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ABSTRACT

Reported are early results of a research program that aims to improve children's mathematics learning by developing attitudes and strategies that support processes of interpretation and meaning construction in mathematics. Considerable research shows that many children learn mathematics as symbol manipulation rules, without meaning. Collaborative problem solving is used as a means for meeting the goals of interpretation and meaning construction. The social setting of shared problem solving provides occasions for modeling effective thinking strategies, critiquing and shaping of thinking, motivation to try new, more active approaches, and scaffolding for an individual learner's initially limited performance. Findings and interpretations from studies with fourth and fifth graders focused on scaffolding are reported, and current and future studies are briefly described. The intimate relationship between conceptual knowledge and problem solving in mathematics presents special constraints for instruction and learning. (MNS)

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MEANING CONSTRUCTION IN MATHEMATICAL PROBLEM SOLVING

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ABSTRACT

This paper reports early results of a program of research that aims to improve children's mathematics learning by developing attitudes and strategies that support processes of interpretation and meaning construction in mathematics. We are examining processes of socially shared problem solving, in which an adult and other children provide scaffolding for individuals' early problem solving efforts. Different ways of scaffolding problem solving efforts and building self-monitoring strategies are explored in early studies. These studies also show that the intimate relationship between conceptual knowledge and problem-solving in mathematics sets special constraints for instruction and learning.

Considerable research now shows that many children learn mathematics as symbol manipulation rules. They do not adequately link formal rules to mathematical concepts--often informally acquired--that give symbols meaning, constrain permissible manipulations, and link mathematical formalisms to real-world situations (Resnick, in press a). Widespread indications of this problem include buggy arithmetic algorithms, algebra malrules, and a general inability to use mathematical knowledge for problem solving. However, hints exist that strong mathematics students are less likely than other students to detach mathematical symbols from their referents. These students seem to use implicit mathematical principles and knowledge of situations involving quantities to construct explanations and justifications for mathematics rules--even when such explanations and justifications are not required by teachers.

This conjecture is supported by research in other fields of learning. For example, it has been shown that good readers are more aware of their own level of comprehension than poor ones; good readers also do more elaboration and questioning to arrive at sensible interpretations of what they read (e.g., Brown, Bransford, Ferrara, & Campione, 1983). Good writers (e.g, Flower & Hayes, 1980), good reasoners in political science and economics (e.g., Voss, Greene, Post, & Penner, 1983), and good science problem solvers (e.g., Chi, Glaser, & Rees, 1982) all tend to treat learning as a process of interpretation, justification and meaning construction. In a few instances intervention programs have improved both the tendency and the ability of students to engage in meaning construction. The best developed line of such research is in the field of reading. Palincsar and Brown (1984), broadly following a Vygotskian analysis of the development of thinking, proposed that extended practice in communally constructing meanings for texts should eventually internalize the meaning construction processes within each individual. Their instructional experiments, in which small groups of children worked cooperatively to interpret a text, showed broad and long-lasting effects on reading comprehension.

We report here on a program of research that is aiming to improve children's mathematics learning by developing attitudes and strategies that support processes of

interpretation and meaning construction in mathematics. Our choice of collaborative problem-solving as a means for meeting this goal reflects an analysis of the nature of cognition that we share with a small, but growing number of psychologists, anthropologists, linguists and sociologists who have been analyzing socially distributed cognition in various applied and school settings (see Resnick, in press c, for a review and interpretation of some of this research).

Socially shared problem-solving sets up several conditions that may be important in the development of mathematical competence. The social setting provides occasions for ***modeling*** effective thinking strategies. Thinkers with more skill (often the instructor, but sometimes more advanced fellow students) can demonstrate desirable ways of attacking problems and constructing arguments. It also permits ***critiquing and shaping*** of thinking because processes of thought as well as results become visible. The social setting is also ***motivating***; through encouragement to try new, more active approaches, and social support even for partially successful efforts, students come to think of themselves as capable of engaging in interpretation and meaning construction. Finally, collaborative problem solving can provide a kind of ***scaffolding*** for an individual learner's initially limited performance. Instead of practicing small bits of thinking in isolation, so that the significance of each bit is not visible, a group solves a problem together. In this way, extreme novices can participate in actually solving the problem and can, if things go well, eventually take over all or most of the work themselves.

INITIAL STUDIES: SPECIFIC KNOWLEDGE AND GENERAL STRATEGIES

Our initial efforts were aimed at examining the extent to which the method of reciprocal teaching, developed by Palincsar and Brown to teach reading comprehension skills, could be applied to mathematics learning. Palincsar and Brown use a highly organized small-group teaching situation, in which children took turns playing the role of teacher, a role in which they pose questions about texts, summarize them, offer clarifications and make predictions. These four activities are thought to induce the kinds of self-monitoring of comprehension that are characteristic of good readers. The adult's role in these sessions, in addition to keeping the general process flowing smoothly, is to model problem-solving processes (including encountering and overcoming difficulties); to provide careful reinforcement for successively better approximations to good meaning construction behaviors on the part of the children; and, above all, to provide scaffolding for the children's problem-solving efforts.

Knowledge-dependence of Mathematical Problem-solving

We began with a series of four sessions with a group of five fifth grade children. In these sessions, word problems involving some aspect of rational numbers were to be solved collaboratively, with children taking turns serving as leader of the discussion. Sessions were tape recorded and full transcriptions prepared. Study of the protocols revealed two fundamental problems that would have to be met in adapting the principles of reciprocal teaching to mathematics. Both are rooted in the fact that mathematics problem-solving is

more strictly knowledge-dependent than is reading.

First, in our problem-solving sessions, children frequently foundered on sheer lack of knowledge of relevant mathematical content--despite our having chosen rational number problems in order to match our sessions' content to what children were studying in their regular mathematics class. This contrasts sharply with conditions in reciprocal teaching groups in reading, where children are rarely outright ***wrong*** in their summaries and questions; their responses may not enchance comprehension very much, but they do not drive it off course, either. An example of the dramatic ways in which insecure basic mathematical knowledge blocked successful problem solving is a situation in which the children had drawn a "pizza" and divided it into six parts, each called "a sixth"; they then shaded three parts, after which they asserted that each shaded part was "a third." In situations like this, the adult must choose between interrupting attention to problem-solving processes to teach basic mathematics concepts and attempting to continue problem-solving with fundamental errors of interpretation. Neither choice seems likely to foster the development of appropriate meaning construction abilities.

Second, part of what makes reciprocal teaching work smoothly in reading is that the same limited set of activities (summarizing, questioning, predicting, clarifying) is carried out again and again. It is not as easy to find repeatable activities of this kind for mathematics, because specific knowledge plays such an important role in solving each problem. We used some very general repeated questions--introduced and repeated by the adult leading the sessions--such as "What is the question we are working on?" "Would a diagram help?" "Does that [answer] make sense?" or "What other problem is like this one?" However, as is also often the case for more mathematically sophisticated Polya-like heuristics, these appeared too general to adequately constrain the children's efforts. For example, they did not know ***what*** diagram to draw (or drew it incorrectly), or could not decide whether an answer was sensible because they had misunderstood basic concepts.

Using Strategies Versus Talking About Them

In a second effort, we attempted to respond to each of these problems in a systematic way. The children were fourth graders; they worked in a group of five for 13 sessions, each led by the same adult. To control for children's lack of specific relevant mathematical knowledge, we chose problems that invoked concepts from the previous year of mathematics instruction rather than the current year. This control for unmastered mathematical content was successful. We encountered very few occasions in which fundamental mathematical errors or lack of knowledge impeded problem solving.

On the basis of cognitive theories of problem solving, we identified four key processes that should be repeated in each new problem-solving attempt. These functions are (1) planning--i.e., analyzing the problem to determine what kinds of procedures are appropriate; (2) organizing the steps for a chosen procedure; (3) carrying out the steps of the procedure; and (4) monitoring each of the above processes to detect errors of sense and of procedure. For each problem to be solved, the four functions were assigned to four different children. The Planner was to take responsibility for leading a discussion of the problem, in order to decide what particular strategies and procedures should be applied. Once a procedure was chosen by the group, the Director's task was to explicitly state the

steps in the procedure. These steps were to be carried out by the Doer at a publicly visible board. The Critic was to intervene whenever an unreasonable plan or an error in procedure was detected.

The tactic of dividing mental problem-solving processes into overt social roles was not initially a success. The research community has shared meaning for terms such as planning, directing and critiquing/monitoring. But, with the exception of the Doer role, these meanings were not conveyed to children by the labels, and we were not successful in verbally explaining them to the children. As a result, the roles became instruments for controlling turn-taking and certain other social aspects of the sessions, but they did not successfully give substantive direction to problem-solving. Children discussed the roles a great deal, but they did not become adept at performing them. This points to a fundamental problem with certain metacognitive training efforts that focus attention on knowledge *about* problem solving rather than on guided and constrained practice in *doing* problem-solving. Such efforts are more likely to produce abilities to talk about processes and functions than to actually perform them.

In session 6, we attempted a modification of one of the roles, the Critic, in order to deal with this problem. The critic's function was distributed to two children, who were each given "cue cards" that they were to use to communicate their criticisms. The cue cards read:

1. "Why should we do that?" [request for justification for a procedure]

2. "Are you sure we should be adding (subtracting, multiplying, dividing)?" [request for justification of a particular calculation]

3. "What are we trying to do right now?" [request for clarification of a goal]

4. "What do the numbers mean?" [insistence that attention focus on meanings rather than calculation and symbol manipulation]

The cue cards served to scaffold the critic function by providing language for a limited set of possible critiques. At first the children used the cue cards more or less randomly and in a rather intrusive fashion. However, during the course of the succeeding seven sessions, children's use of the cue cards became more and more refined, so that they used them on appropriate occasions and in ways that enhanced rather than disrupted the group's work.

CURRENT AND PLANNED STUDIES

In studies currently underway and planned, we are examining more restricted forms of shared problem-solving, in order to gain greater experimental and analytical control. We will study groups engaged in collaborative solution of various classes of mathematics problems. We will also study groups whose task is to construct story situations that could generate particular arithmetic expressions or equations (cf. Resnick, Cauzinille & Mathieu, in press; Putnam, Lesgold, Resnick, and Sterret, this volume). Finally, we will study

groups whose task is to instruct new (to them) mathematical procedures and algorithms.

Planning and Means-Ends Analysis

A study currently underway examines pairs of children solving problems that are particularly suited to classical "means-ends" problem-solving strategies (cf. Newell & Simon, 1972). Participants in the study were 12 pairs of children, 3 pairs each in grades 4, 5, 6 and 7. Each pair of children met three times for 40 minutes and solved two to six problems.

To scaffold the means-end problem-solving strategy, children were given a Planning Board to work with. The board provides spaces for recording what is known (either given in the problem statement or generated by the children) and what knowledge is needed (goals and subgoals of the problem). Using the board, children can work both "bottom-up" (generating "what we know" entries) or "top-down" (generating "what we need to know" entries). A space at the bottom is provided for calculation. Each child writes with a different color pen, so that we can track who is responsible for each entry. Full verbal transcripts of each session are also prepared.

At each grade level, one pair of children was assigned to each of three conditions. The conditions were:

1. Planning Board With Maximum Instruction. The children solved problems using the planning board. During the first session, the adult demonstrated use of the planning board, and then participated in the first two sessions as a provider of hints and prompts to further scaffold the problem solving process and the use of the board.

2. Planning Board With Minimum Instruction. The children solved problems using the planning board. The adult demonstrated the board and provided hints and prompts during the first session only.

3. Control. The children solved problems without the planning board during all three sessions.

Preliminary inspection of the data suggest that older children and children with more training come to use the board more efficiently. They also generated more goals and inferences on the board. However, in three sessions, there was no effect on accuracy of solutions.

Protocols of the sessions are now being coded in a form that allows us to plot the logical structure of the joint problem-solving effort--i.e., what goals are generated and in what order, what inferences are made from data that is given in the problem statement, how what is known is mapped to goals. Our coding will also permit us to examine the nature of the social sharing of the problem-solving effort. For example, we will be able to determine whether the two children work together on a particular goal or whether they work in parallel; and whether role specializations arise, such as one child working "bottom up" and the other "top down."

GENERAL DISCUSSION

The use of a social setting for practicing problem solving is shared by a number of other investigators, including some in the field of mathematics (see Resnick, in press b). Lampert (1986) conducts full-class discussion in which children invent and justify solutions to mathematical problems. Lampert's discussions are like those of reciprocal teaching in that they are carefully orchestrated by the teacher, and include considerable modeling of interpretive problem-solving by the teacher. Schoenfeld's (1985) work with college students shares many features of the Lampert class lessons, but with considerably more focus on overt discussion of general strategies for problem solving than Lampert uses. Lesh (1982), by contrast, shares reciprocal teaching's small-group format for collaborative problem solving, but has no teacher present. This means that Lesh's problem solving groups benefit from the debate and mutual critiquing that children give each other, but do not have the opportunity to observe expert models engage in the process and are not taught any specific techniques for problem analysis or solution. Scaffolding will also be limited to what children are able to provide spontaneously for one another. The kinds of analyses that we are developing for our data could also be applied to problem-solving groups functioning in these alternative modes. Eventually, comparative studies should help us understand more how these alternative approaches to collaborative problem-solving actually function in supporting and developing mathematical competence.

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